

Masses of multiquark droplets

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Abstract

The mass formulae for finite lumps of strange quark matter with u , d and s quarks, and non-strange quark matter consisting of u and d quarks are derived in a non-relativistic potential model. The finite-size effects comprising the surface, curvature and even, the Gauss curvature were consistently obtained, which shows a converging trend. It is found that there is a possibility for the formation of metastable strangelets of large mass. The model predicts low charge to mass ratio as the characteristic signature of strange matter in agreement with the relativistic studies. This study also yields an independent estimate for the bag energy density B , which is in agreement with the M.I.T bag model value.

I. INTRODUCTION

The possibility of strange quark matter to be the ground state of QCD and the probable stability of such matter in finite lumps called as strangelets, speculated by many authors [1-5] has generated a spurt of activity during the last few years. The general framework adopted for such studies has been the relativistic Fermi gas model with relevant QCD parameters like the bag constant B , strong coupling constant α_c and the strange quark mass m_s . Due to large uncertainty in these parameters, it has not yet been possible to arrive at a conclusive result.

The calculations of Chin and Kerman [3] and Freedman and McLerran [4], which includes the lowest order quark-quark interaction shows that infinite quark matter even with strangeness is unstable against the nucleon decay. Regarding the probable stability of a finite lump of quark matter, Farhi and Jaffe [5] were the first to consider the mass of the strangelets by adding the surface energy and the Coulomb energy terms to the volume energy, by initially treating the surface tension coefficient as a parameter. Later Berger and Jaffe [6] developed a formula to calculate the surface tension coefficient. They showed that surface energy is zero for massless case, and to be about 100 MeV for the massive case [$m_u = m_d = 0, m_s \simeq 150 \text{ MeV}$]. In this work, Berger and Jaffe have dropped the quark-quark interaction with the supposition that it can be absorbed into the bag constant parameter. Thus, they have developed a mass formula assuming a system of non-interacting quarks confined in a finite-size cavity for which they included the surface and the Coulomb effects.

Recently, following Mardor and Svetitsky [7] who have pointed out that the curvature energy might play a decisive role for the stability of strangelets, Madsen [8] has obtained an estimate for the curvature energy coefficient to be about 435 MeV for the massless case. This has dramatic consequences for the stability of low mass strangelets. In view of its importance, the curvature coefficient should also be determined for the massive case, which has not yet been derived. The indication of predominance of the curvature over the surface term is already intriguing. Therefore, it is highly desirable to develop a mass formula with surface, curvature and Gauss curvature terms correctly calculated showing convergence trend. During the last two decades, extensive application of non-relativistic potential models [9] to the description of ground-state, excited-state and spectroscopic properties of baryons and mesons have been amazingly successful. In view of this, and due to the momentous nature of the problem of stability of strangelets and its far reaching consequences, it is worthwhile to attempt at a development of a mass formula based on a non-relativistic approach. In this paper, we report on such an attempt, which has the advantage of reliable estimation of the various coefficients with practically no parameter except the baryon number density.

In Sec.II, we present the details of our non-relativistic formalism used to derive the mass formulae for multiquark droplets including the finite-size effects. We then discuss the results obtained in our model in Sec.III. Finally, we summarize and present our conclusions in Sec.IV.

II. FORMALISM

In this work, similar to Berger and Jaffe [6] relativistic study, we consider a system of finite number of non-interacting particles confined in a box. This can be represented by an infinite square well potential for which the correct enumeration of quantum states has been done by Hill and Wheeler [10] who obtained the density of states for such a system as

$$dN = \frac{gV}{2\pi^2} k^2 dk - \frac{gS}{8\pi} k dk + \frac{gL}{8\pi} dk \quad (1)$$

where g is the degeneracy factor. For a spherical box of radius R ; $V = \frac{4}{3}\pi R^3$, $S = 4\pi R^2$ and $L = 2\pi R$. It is then straight forward to obtain an expression for the total number of particles N as

$$N = \int_0^{k_F} \frac{dN}{dk} dk = \frac{gV}{6\pi^2} k_F^3 - \frac{gS}{16\pi} k_F^2 + \frac{gL}{8\pi} k_F \quad (2)$$

and for the total kinetic energy E_{kin} as

$$E_{kin} = \int_0^{k_F} \frac{\hbar^2 k^2}{2m} \frac{dN}{dk} dk = \frac{\hbar^2}{2m} \left[\frac{gV}{10\pi^2} k_F^5 - \frac{gS}{32\pi} k_F^4 + \frac{gL}{24\pi} k_F^3 \right] \quad (3)$$

where k_F is the Fermi momentum and m is the mass of the particles. Considering a system of one kind of quarks, we have $g = 6$. We then express the Fermi momentum k_F in terms of the radius R , by inverting Eq.(2) and retaining terms upto $O(N^{-1})$ as

$$k_F \simeq (\pi^2 \rho)^{1/3} [1 + C_1 (\pi^2 \rho)^{-1/3} + C_2 (\pi^2 \rho)^{-2/3} + C_3 (\pi^2 \rho)^{-1}]^{1/3} \quad (4)$$

where $C_1 = 3\pi S/(8V)$, $C_2 = 2C_1^2/3 - 3\pi L/(4V)$, $C_3 = C_1^3/3 - C_1 3\pi L/(4V)$, and the total particle number density $\rho = N/V$. Then, substituting the above expression for k_F in Eq.(3), and grouping all the terms with same powers of N , we arrive at the following expression for E_{kin} upto the order of N^0

$$E_{kin} = \frac{\hbar^2}{2m} \left[\frac{3\pi^{4/3}}{5} N + \left(\frac{3}{4}\right)^{5/3} \pi^2 N^{2/3} + \left(\frac{9\pi^3}{16} - \pi^2\right) \left(\frac{3}{4\pi}\right)^{1/3} N^{1/3} + \frac{9\pi^{4/3}}{16} \left(\frac{5\pi^2}{8} - 2\pi\right) \right] \rho^{2/3} \quad (5)$$

The coefficient of N , $N^{2/3}$, $N^{1/3}$ and N^0 are termed as volume, surface, curvature and Gauss curvature contributions respectively to the total kinetic energy.

Now, in order to obtain an expression for the total mass of a multiquark system, the following important effects are to be included.

i) For the system to be stable at finite density, pressure has to be balanced at the surface. To ensure this, we have to add an extra term BV which is quite analogous to the MIT bag term [11].

ii) The center of mass correction for a many-particle system can be taken into account in an approximate way by multiplying the factor $\hbar^2/2m$ in the kinetic energy expression by a factor $(\frac{N-1}{N})$, as customarily done in the non-relativistic Hartee-Fock calculations [12].

iii) In addition to the finite-size effects, the Coulomb effect has to be included. For a spherical system of radius R and charge Ze , the Coulomb energy is given by $E_{Coul} = \frac{3Z^2e^2}{5R}$.

iv) Lastly, the rest mass energy of all the particles ($= mN$) confined in the spherical box must be included.

Thus, including all the above effects, the total mass of a N particle system is given in a compact form as

$$M(N) = E_{kin} + BV + E_{Coul} + mN. \quad (6)$$

We then eliminate the bag energy density B by using the stability condition, which is nothing but the pressure balance equation

$$\left. \frac{d(M/N)}{d\rho} \right|_{\rho_{oq}} = 0. \quad (7)$$

where ρ_{oq} is the equilibrium quark number density. This gives,

$$B = \frac{2}{3} \frac{E_{kin}}{N} + \frac{1}{3} \frac{E_{Coul}}{N} \quad (8)$$

Then, in the bulk limit, neglecting the Coulomb effects, we can rewrite B as

$$B = \frac{2}{3} \left[\frac{\hbar^2}{2m} \frac{3\pi^{4/3}}{5} \right] \rho_{oq}^{2/3}. \quad (9)$$

Thus, we arrive at a formula for the bag energy density B which can be computed using the mass of the constituent quarks and the number density of the system.

It is interesting to recall here that the usual bag models of hadrons employs a bag constant B , which is considered an universal constant of nature. It is being widely used in the study of hadron properties and the phase transition of hadronic matter to quark-gluon-plasma. Normally, the value of B has been determined from the ground state properties of hadrons, which lies between 58 MeV fm^{-3} [$B^{1/4} = 145 \text{ MeV}$] and 86 MeV fm^{-3} [$B^{1/4} = 160 \text{ MeV}$].

On the nonrelativistic front, potential model studies have also been very successful in describing the baryon properties. Therefore, the above formula for B given by Eq.(9) is most likely to give an independent realistic estimate of the same. We then express the quark number density ρ_{oq} in terms of the baryon number density ρ_o as $\rho_o = \rho_{oq}/3$ and fix the mass of the u -quark to be 300 MeV which is normally used in non-relativistic models [13]. We take the value of ρ_o to be about $1.3\rho_{nuc}$, where the nuclear matter density ρ_{nuc} is given by 0.153 fm^{-3} [14]. Using these values, we obtain our value of B to be about 83 MeV fm^{-3} . Amazingly, this value lies within the range of relativistic values of about 58 MeV fm^{-3} and 86 MeV fm^{-3} . This result gives us confidence in the reliability of our

model for the study of stability of multi-quark droplets. Further, the above formula given by Eq.(9), clearly brings out the dependence of the bag energy density B on the baryon number density and the quark mass.

Finally, we now obtain the following mass formula for a system of N particles, each of mass m and charge q , by substituting the value of B given by Eq.(8) in Eq.(6).

$$M(N) = \frac{5}{3}E_{kin} + \frac{4}{3}E_{Coul} + mN \quad (10)$$

A. Two flavour system.

The mass formula given by Eq.(10) is for a system consisting of one kind of quarks. It can be generalised to obtain the mass of a system consisting of quarks of more than one flavour. Here, we apply it to a multi-quark droplet consisting of u and d quarks, hereafter referred to as “udilet”. For a given number of u and d quarks N_u and N_d respectively, we define an asymmetry parameter $\delta = (N_d - N_u)/N_q$, where $N_q = N_u + N_d$. So, the quark density for each flavour can be expressed in terms of δ and ρ_{oq} as

$$\begin{aligned} \rho_u &= \frac{\rho_{oq}}{2}(1 - \delta) \\ \rho_d &= \frac{\rho_{oq}}{2}(1 + \delta) \end{aligned} \quad (11)$$

with $\rho_{oq} = N_q/V$. The total kinetic energy for a two flavour system is obtained by summing over the respective contributions, given by Eq.(5) for u and d quarks. Then, one can obtain the total mass of an udilet in terms of the baryon number density $\rho_o [= \rho_{oq}/3]$ using Eqs.(10) and (11) as

$$M_2(A) = \frac{5}{3} \frac{\hbar^2}{2m_u} \frac{3\pi^{4/3}}{5} [\rho_u^{5/3} + \rho_d^{5/3}] \frac{4\pi R^3}{3}$$

$$\begin{aligned}
& + \frac{5}{3} \frac{\hbar^2}{2m_u} \frac{3\pi^{5/3}}{16} [\rho_u^{4/3} + \rho_d^{4/3}] 4\pi R^2 \\
& + \frac{5}{3} \frac{\hbar^2}{2m_u} \left[\frac{9\pi^2}{32} - \frac{\pi}{2} \right] [\rho_u + \rho_d] 2\pi R \\
& + \frac{5}{3} \frac{\hbar^2}{2m_u} \frac{9\pi^{4/3}}{16} \left[\frac{5\pi^2}{8} - 2\pi \right] [\rho_u^{2/3} + \rho_d^{2/3}] \\
& + \frac{4}{3} \left[\frac{3Ze^2}{5R} \right] + 3m_u A
\end{aligned} \tag{12}$$

where we have used $m_d = m_u$ and $A = N_q/3$. By substituting for ρ_u , ρ_d and the radius $R = r_o A^{1/3}$ with $\frac{4}{3}\pi r_o^3 \rho_o = 1$, we thus arrive at the Bethe-Weizsacker like mass formula for the udilets upto the order of A^{-1} and δ^2 .

$$\frac{M_2(A)}{A} = a_{v2} + a_{s2} A^{-1/3} + a_{c2} A^{-2/3} + a_{o2} A^{-1} + a_z Z^2 A^{-4/3} + 3m_u \tag{13}$$

where a_{v2} , a_{s2} etc are defined as $a_{v2} = a_v(1 + \frac{5}{9}\delta^2)$, $a_{s2} = a_s(1 + \frac{2}{9}\delta^2)$, $a_{c2} = a_c$, $a_{o2} = a_o(1 - \frac{1}{9}\delta^2)$, and the various coefficients are given by,

$$\begin{aligned}
a_v &= \frac{5}{3} \frac{\hbar^2}{2m_u} \frac{9\pi^{4/3}}{5} (1.5)^{2/3} \rho_o^{2/3} \\
a_s &= \frac{5}{3} \frac{\hbar^2}{2m_u} \left(\frac{3}{4}\right)^{5/3} \left(\frac{3}{2}\right)^{4/3} 2\pi^2 \rho_o^{2/3} \\
a_c &= \frac{5}{3} \frac{\hbar^2}{2m_u} \left[\frac{9\pi}{16} - 1 \right] 3\pi^2 \left(\frac{3}{4\pi}\right)^{1/3} (1.5)^{2/3} \rho_o^{2/3} \\
a_o &= \frac{5}{3} \frac{\hbar^2}{2m_u} \left[\frac{9\pi^{4/3}}{8} \left[\frac{5\pi^2}{8} - 2\pi \right] - \frac{6\pi^{4/3}}{5} \right] \rho_o^{2/3} \\
a_z &= \frac{4}{3} \left[\frac{3e^2}{5r_0} \right]
\end{aligned} \tag{14}$$

B. Three flavour system

Similar to the two flavour system discussed above , we now consider a multi-quark droplet consisting of u , d and s quarks usually referred to as strangelet. For this case of three flavour system, we need to define two asymmetry parameters namely, $\delta_{ud} = (N_d - N_u)/N_q$ and $\delta_{us} = (N_s - N_u)/N_q$, where $N_q = N_u + N_d + N_s$. We can then express the quark density of each flavour in terms of δ_{ud} , δ_{us} and ρ_{oq} as

$$\begin{aligned}\rho_u &= \frac{\rho_{oq}}{3}(1 - \delta_{us} - \delta_{ud}) \\ \rho_d &= \frac{\rho_{oq}}{3}(1 - \delta_{us} + 2\delta_{ud}) \\ \rho_s &= \frac{\rho_{oq}}{3}(1 + 2\delta_{us} - \delta_{ud}).\end{aligned}\tag{15}$$

The total kinetic energy for this system is obtained by summing over the respective contributions given by Eq.(5) for u , d and s quarks. Using Eqs.(10) and (15), we thus arrive at the following mass formula for strangelets in terms of the baryon number A and its density ρ_o retaining, terms upto $O(A^{-1})$ and second order in the asymmetry parameters.

$$\frac{M_3(A)}{A} = a_{v3} + a_{s3}A^{-1/3} + a_{c3}A^{-2/3} + a_{o3}A^{-1} + a_z Z^2 A^{-4/3} + (f_u + f_d)m_u + f_s m_s \tag{16}$$

where the quark fractions can be expressed using Eq.(15), as follows .

$$\begin{aligned}f_u &= \frac{N_u}{A} = 1 - \delta_{us} - \delta_{ud} \\ f_d &= \frac{N_d}{A} = 1 - \delta_{us} + 2\delta_{ud} \\ f_s &= \frac{N_s}{A} = 1 + 2\delta_{us} - \delta_{ud}\end{aligned}\tag{17}$$

The various coefficients a_{v3}, a_{s3} etc in Eq.(16) are given by,

$$\begin{aligned}
a_{v3} &= \frac{5}{3}c_v\left[\left(\frac{2}{m_u} + \frac{1}{m_s}\right) + \left(\frac{1}{m_u} - \frac{1}{m_s}\right)\frac{5}{3}(\delta_{ud} - 2\delta_{us}) + \frac{5}{9}\left(\frac{\Delta_1^2}{m_u} + \frac{\Delta_2^2}{m_s}\right)\right] \\
a_{s3} &= \frac{5}{3}c_s\left[\left(\frac{2}{m_u} + \frac{1}{m_s}\right) + \left(\frac{1}{m_u} - \frac{1}{m_s}\right)\frac{4}{3}(\delta_{ud} - 2\delta_{us}) + \frac{2}{9}\left(\frac{\Delta_1^2}{m_u} + \frac{\Delta_2^2}{m_s}\right)\right] \\
a_{c3} &= \frac{5}{3}c_c\left[\left(\frac{2}{m_u} + \frac{1}{m_s}\right) + \left(\frac{1}{m_u} - \frac{1}{m_s}\right)(\delta_{ud} - 2\delta_{us})\right] \\
a_{o3} &= \frac{5}{3}c_o\left[\left(\frac{2}{m_u} + \frac{1}{m_s}\right) + \left(\frac{1}{m_u} - \frac{1}{m_s}\right)\frac{2}{3}(\delta_{ud} - 2\delta_{us}) - \frac{1}{9}\left(\frac{\Delta_1^2}{m_u} + \frac{\Delta_2^2}{m_s}\right)\right]
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
c_v &= \frac{\hbar^2}{2} \frac{3}{5} \pi^{4/3} \rho_o^{2/3} \\
c_s &= \frac{\hbar^2}{2} \left(\frac{3}{4}\right)^{5/3} \pi^2 \rho_o^{2/3} \\
c_c &= \frac{\hbar^2}{2} \left[\frac{9\pi^3}{16} - \pi^2\right] \left(\frac{3}{4\pi}\right)^{1/3} \rho_o^{2/3} \\
c_o &= \frac{\hbar^2}{2} \left[\frac{9\pi^{4/3}}{16} \left(\frac{5\pi^2}{8} - 2\pi\right) - \frac{3\pi^{4/3}}{5}\right] \rho_o^{2/3} \\
\Delta_1^2 &= (\delta_{ud} + \delta_{us})^2 + (2\delta_{ud} - \delta_{us})^2 \\
\Delta_2^2 &= (2\delta_{us} - \delta_{ud})^2.
\end{aligned} \tag{19}$$

III. RESULTS AND DISCUSSIONS.

The mass formulae given by Eqs.(13) and (16) are most general, which can be used for any arbitrary combination of u, d and s quarks. To calculate the various coefficients like volume, surface *etc*, we have chosen the mass of u and d quarks to be 300 MeV which lies within the range of values normally used in the non-relativistic potential model calculations [13] of baryon spectroscopy. We take the mass of s quark to be 400 MeV , which is quite reasonable. The baryon number density ρ_o is assumed to be about 1.3

times the nuclear matter density $\rho_{nuc} = 0.153 fm^{-3}$. For the sake of convenience, we calculate these coefficients by imposing charge neutrality condition, which for udilets implies $N_d = 2N_u$, $\delta = \frac{1}{3}$ and for strangelets $N_u = N_d = N_s$, $\delta_{ud} = 0, \delta_{us} = 0$. The value of the coefficients so computed for udilets and strangelets are given in Table I. It is satisfying to note that the contribution of the various terms successively decreases leading to a faster convergence of $M(A)/A$. It is interesting to note that the Gauss curvature term has negative sign and so has the opposite effect on the mass compared to surface and curvature terms. This is in fact is a welcome feature. It must be realised that these kind of mass formulae based on Fermi gas approximation are not valid for very small baryon numbers. Because of its general power series structure in $A^{-1/3}$, the higher order contributions becomes progressively important for low A values. However, we feel these formulae are more appropriate for baryon number $A > 50$ as also noted by Jaffe [15]. In Table II, we have presented the corresponding masses of udilets and strangelets for certain typical values of A . From symmetry considerations, strangelets are expected to be more stable than the corresponding udilets for a given baryon number. This is borne out quite well in our calculations as can be seen from Table II. The energy per baryon for strange quark matter is about $1275 MeV$, and for non-strange quark matter consisting of u and d quarks, it is about $1320 MeV$. Thus, both these quark matters are unstable against nucleon decay. So, we find in this model the nuclear matter to be the ground state of hadronic matter. However, this does not *apriori* preclude the possibility of meta-stable strangelet formation. Before, we embark upon the investigation of the possibility of strangelet formation in this model, we would like to consider the influence of the baryon

number density ρ_o and strangeness on the relative stability of strangelets and udilets.

In Fig.1, we illustrate the effect of ρ_o on the relative stability between the strangelets and udilets, using the general mass formula for strangelets and udilets given by Eqs.(16) and (13). We have found the most flavour-stable isobar, by minimising the mass with respect to δ for udilets, and δ_{ud} and δ_{us} for strangelets. In Fig.1a, the difference between the masses of the strangelets and udilets so obtained are plotted as a function of ρ_o/ρ_{nuc} , for a typical baryon number $A = 400$. It can be seen that mass of the strangelets is lower than the corresponding udilets for all densities. The minimum difference between their masses is about 4 MeV. This feature is true for any baryon number A is shown in Fig.1b, where the surface and Coulomb effects are neglected

In Fig.2, we illustrate the effect of strangeness on the relative stability between the strangelets and udilets for the same $A = 400$. Using the general formula for strangelets given by Eq.(16), we have determined for a given δ_{us} , the flavour-stable value of mass $M_3(A)$ by minimizing the total mass with respect to δ_{ud} . For two chosen values of ρ_o , namely $1.27 \rho_{nuc}$ and $2.14 \rho_{nuc}$, the results so obtained are presented as solid curves in Fig.2 as a function of f_s , where f_s in terms of δ_{ud} and δ_{us} is given by Eq.(17). It is interesting to find that the mass shows a well defined minimum for f_s at about 0.7. Then to show the relative stability with respect to udilets having the same baryon number A , we have minimised the mass $M_2(A)$ given by Eq.(13) with respect to δ . We have represented the most stable mass of udilets so obtained for the same two values of ρ_o as dashed lines. It can be seen from Fig.2 that at the optimum value of f_s , the difference between the mass of strangelets and udilets is maximum, which gradually decreases and tends to zero

as f_s is varied about the optimum value. Therefore, there is a maximum value of f_s , referred to as f_s^m , above which udilets becomes more stable than the strangelets. Thus, there is a ‘window’ in f_s space, only within which strangelets are more stable than the corresponding udilets. This window region depends upon density and increases as ρ_o is increased, which implies that hyper strange multiquark droplets are more probable at higher densities. This is shown in Fig.3, where f_s^m is plotted as a function of ρ_o/ρ_{nuc} for the same value of A , namely $A = 400$. It can be seen that as the density increases, f_s^m increases. The shaded portion in Fig.3 represents the region where strangelets are more stable relative to udilets, and the unshaded region represents the vice-versa.

Now, we would like to investigate the probable stability of strangelets in this model. For a given δ_{us} , we determine the most stable mass per baryon $M_V(A)/A$ of the strangelet in the bulk limit, by minimizing the mass given by Eq.(16) with respect to δ_{ud} . In the absence of Coulomb interaction $N_u = N_d$; $f_s(= N_s/A)$ is determined by using these values of δ_{ud} and δ_{us} in Eq.(17). Following Chin and Kerman, we plot in Fig.4 $M_V/A - M_N$ versus f_s , where the masses of the nucleon, lambda, cascade and omega are shown as solid dots. It can be seen that strangelets are unstable against nucleon and lambda decay even at the optimum value of f_s , which is about 0.7. As the optimum value of f_s is less than unity, the number of s quarks is less than the number of u and d quarks. Therefore, the excess of the u and d quarks over that of s quarks, will occupy the higher momentum states than the s quarks. These u and d quarks will then form nucleons and strongly decay, analogous to the nucleon dripping from a highly radioactive nucleus. At this stage, as the mass of the lambda is higher than that of the nucleon, we expect the nucleon decay

to be more probable than the lambda particle. Thus, the system is likely to be driven towards a state of greater strangeness, where it is stable against cascade and omega decay, as per our calculations. Under these conditions, strangelets can be formed in a metastable state. It can only decay by weak leptonic decay process. This possibility gives us some hope of detecting strangelets, but only of very large mass.

In the present model, we have the density as the only free parameter, because the masses of u , d and s quarks are constrained by the non-relativistic potential model studies of baryon spectroscopy. We find that with the increase of density, the masses of the quarks have to be lowered in order to make the system stable against omega emission. For a given density, the masses of these quarks can be varied within the acceptable limits to ensure the same. In Fig.5, fixing the density at about $1.3 \rho_{nuc}$, we have shown the range of m_u and $m_s - m_u$ for which the omega baryon is bound in our model. It is interesting to note that the mass of s quark comes about to be order of 100 MeV higher than the u and d quark masses, which is quite reasonable. Thus, we find that there is a sizable range of these parameters in which metastable strangelet formation is allowed.

As found by Farhi and Jaffe [5] in the relativistic study of strange matter, a possible signature for strange matter is its characteristic low charge to mass ratio, unlike in the case of ordinary nuclei, where $Z \simeq A/2$. We would like to investigate whether our model would corroborate their finding. So, for the case of a strangelet of baryon number A , we obtain the flavour-stable value of the charge number Z_3^* given by

$$Z_3^* = -A[\delta_{us}^* + \delta_{ud}^*]$$

where δ_{us}^* and δ_{ud}^* are determined by simultaneously the two equations,

$$\frac{dM_3/A}{d\delta_{us}} \Big|_{\delta_{us}^*} = 0$$

$$\frac{dM_3/A}{d\delta_{ud}} \Big|_{\delta_{ud}^*} = 0$$

using Eq.(16).

Similarly, the flavour-stable value of the charge number Z_2^* for the corresponding udilet is obtained using

$$Z_2^* = \frac{A}{2}[1 - 3\delta^*].$$

with δ^* determined from the condition

$$\frac{dM_2/A}{d\delta} \Big|_{\delta^*} = 0$$

where, the mass of the uilets $M_2(A)$ is given by Eq.(13).

In Table III, we have presented the values of Z_2^* and Z_3^* obtained in the above calculation for various baryon number A . For the sake of comparison we have also given the value $Z = A/2$, which is appropriate for nuclei of large A . It can be seen that the Z_{min} for the uilets (Z_2^*) is somewhat lower than the corresponding nuclei, which becomes substantial with the increase of A . However, for the strangelets the Z_{min} value (Z_3^*) is strikingly low. It is indeed satisfying to find that the present non-relativistic study arrives at a similar conclusion as that of the relativistic study of Farhi and Jaffe. Thus, the low charge to mass ratio as a strong signature of strangelets is being reinforced by our present study.

In summarising our results and discussions, we find that the present non-relativistic model has surprisingly succeeded in describing most of the features of strangelets as

found in the relativistic studies. This success and the reliability of our model can be solely attributed to the sound foundation based on the following two elements, due to a which straight forward calculation was possible:

- (i). The confinement of quarks is best represented by the infinite square well potential.
- (ii). For this potential, the exact enumeration of quantum states was given by Hill and Wheeler, which has been used.

IV. CONCLUSION

In conclusion, we have developed the mass formulae for finite lump of quark matter for systems with u and d quarks, and also for systems with u , d and s quarks, in a non-relativistic model. The complete and consistent mass formula including all finite-size effects comprising the surface, curvature, and even the Gauss curvature has been derived for both the strangelets and udilets. For the first time, we have obtained an estimate of the Gauss curvature term. Further, this model study provides an independent estimate of the bag energy density B , which we find to be in agreement with the M.I.T bag model value.

We have shown that for a given baryon density there exists an upper bound on the strangeness below which strangelets are more stable than the udilets. This upper limit on the number of s quarks increases as density increases, which implies that hyperstrange strangelets are more probable at higher densities.

Finally, our calculation shows that there is a finite possibility for the formation of metastable strangelets of very large mass. The present study also predicts low charge to mass ratio as the characteristic signature of strange matter in agreement with the relativistic studies.

The success of this model is due to the use of infinite square well potential, which adequately describes the confinement of quarks and for which exact quantum density of states can be obtained.

References

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FIGURE CAPTIONS

FIG.1. The difference between the mass per baryon of strangelets and udilets is plotted against the baryon number density ρ_o in the units of the nuclear matter density ρ_{nuc} ; (a) for $A = 400$ and (b) in the bulk limit neglecting the Coulomb effects.

FIG.2. The mass per baryon of strangelets relative to the nucleon mass is plotted against the s quark fraction f_s as solid curves for two chosen values of baryon density ρ_o , for a given baryon number $A = 400$. The upper curve corresponds to $\rho_o = 2.14\rho_{nuc}$ and the lower one to $1.27\rho_{nuc}$. The corresponding mass of udilets relative to the nucleon mass is shown as dashed lines.

FIG.3. The maximum value of the s quark fraction f_s^m is plotted against the baryon density ρ_o in units of the nuclear matter density ρ_{nuc} , for a given baryon number $A = 400$. The dotted region represents where the strangelets are more stable than the corresponding udilets.

FIG.4. The mass per baryon of the strangelets M_v/A relative to the nucleon mass in the bulk limit (neglecting the Coulomb effects) is plotted against the s quark fraction f_s . The masses of the nucleon (M_N), lambda particle (M_Λ), cascade particle (M_Ξ) and omega particle (M_Ω) are shown as solid dots.

FIG.5. The range of the parameters m_u and m_s , the mass of u and s quarks respectively favouring stability of strange quark mass against omega emission. The contour plot of the mass per baryon M_v/A of the strangelets with $f_s = 3$ (see Eq.17) in the bulk limit (neglecting the Coulomb effects) is plotted against m_u and difference between m_u and m_s . The corresponding numerical values of M_v/A are shown .

TABLE CAPTIONS

Table I. The values of the volume, surface, curvature and Gauss curvature coefficients, a_v , a_s , a_c and a_o respectively obtained using the charge neutrality condition are given for both the udilets and strangelets.

Table II. The mass per baryon for strangelets (M_3/A) and udilets (M_2/A) calculated using the charge neutrality condition are given for various baryon number A .

Table III. The flavour-stable value of the charge number for udilets (Z_2^*) and strangelets (Z_3^*) are given for various baryon number A .

Table I

<i>System</i>	a_v (MeV)	a_s (MeV)	a_c (MeV)	a_o (MeV)
udilett (udd)	417.53	779.14	510.59	-286.77
strangelett (uds)	275.10	608.89	468.04	-304.67

Table II

A	M_2/A (MeV)	M_3/A (MeV)
50.00	1560.90	1468.77
100.00	1506.22	1424.96
200.00	1464.25	1391.38
300.00	1444.35	1375.48
500.00	1423.22	1358.63
1000.00	1400.26	1340.36
3000.00	1373.91	1319.46
5000.00	1364.78	1312.25
10000.00	1354.76	1304.34

Table III

A	$Z = A/2$	Z_2^*	Z_3^*
300.00	150.00	103.79	28.78
500.00	250.00	152.78	44.27
1000.00	500.00	246.42	74.38
3000.00	1500.00	470.38	147.40
5000.00	2500.00	609.49	193.17
10000.00	5000.00	837.78	268.86
50000.00	25000.00	1588.38	521.26
100000.00	50000.00	2041.08	674.98
1000000.00	500000.00	4503.32	1516.66